



Advanced Energetic Particle Transport Model

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- Predicting the dynamics of a **burning plasma** on **long time scales**, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g., **ITER**;
- the crucial role of **energetic particles** [Zonca et al. 2015](#); [Chen and Zonca 2016](#), must be properly described;



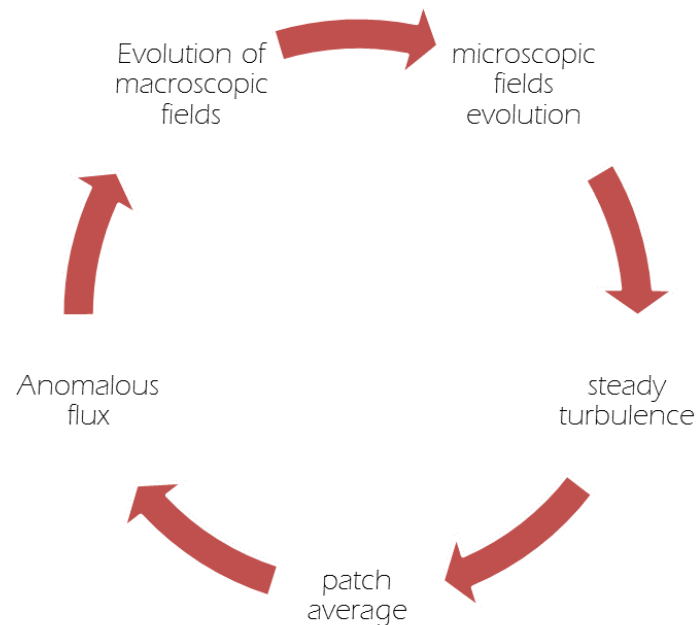
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- the crucial role of **energetic particles** Zonca et al. 2015; Chen and Zonca 2016, must be properly described;
- a **first-principle-based**, self-consistent approach is crucial;
- extending **gyrokinetic simulations** to these time scales is a challenging task from the computational resource point of view, i.e., $\sim 10^{24}$ grid points;
- **simplifying assumptions** based on physics understanding and **first principles** must be introduced;



- Transport equations define the evolution of **radial profiles**

$$\langle \partial_t n \rangle_\psi = - \langle \nabla \cdot (n \mathbf{V}) \rangle_\psi ;$$

- consistent with a **slowly evolving** ($\omega^{-1} \partial_t \log p_0 \sim \delta^2$) **equilibrium** distribution function;

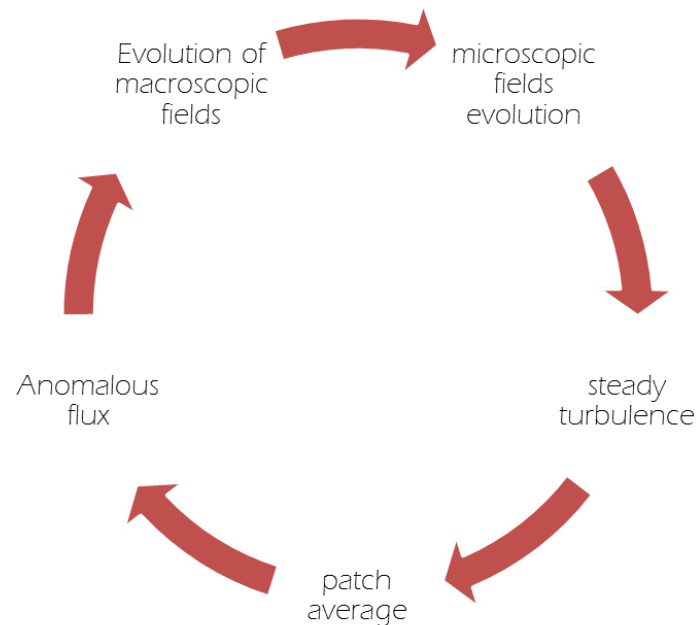




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- consistent with a **slowly evolving** ($\omega^{-1} \partial_t \log p_0 \sim \delta^2$) **equilibrium** distribution function;
- Implicit **separation of scales** between equilibrium and fluctuations;
- Local **Maxwellian** is assumed;



Need for generalization!!!



We aim at 1. providing the **general expressions** describing **EPs** (plasma) dynamics on long time scales (**transport**) and 2. introducing a framework to solve these equations within different levels of **reduced dynamics**.

By means of this approach it will be possible to:

- define the concept of **nonlinear equilibrium**;
- describe the physics of **burning plasmas** where alpha particles will play a **key role in transport studies** by interacting with thermal components;
- the derivation, see [Falessi 2017](#); [Falessi and Zonca 2019](#), is based on the **Phase space zonal structures** theory, see [Chen and Zonca 2016](#).



- Coupling between fluctuating fields can generate **zonal flows** and **zonal fields**;
- **Crucial elements for regulating turbulent fluxes**, e.g., by scattering instability turbulence to shorter radial wavelength stable domain...

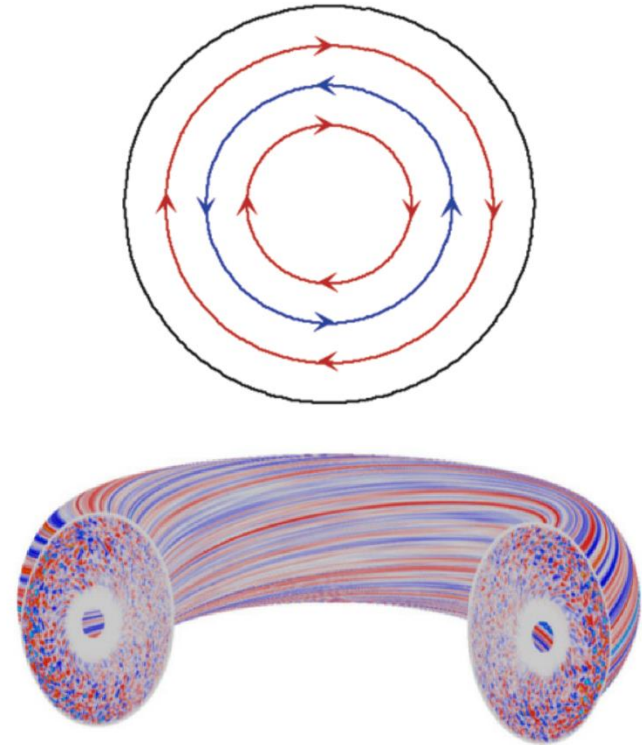


Figure: Courtesy of Y. Xiao et al., PoP 2015.



- Coupling between fluctuating fields can generate **zonal flows** and **zonal fields**;
- **Crucial elements for regulating turbulent fluxes**, e.g., by scattering instability turbulence to shorter radial wavelength stable domain...
- analogously, **zonal structures** in the **density and temperature profiles** are unaffected by rapid collision-less dissipation;
- collision-less undamped fluctuations in the phase space are called **phase space zonal structures** [Chen and Zonca 2016](#), [Falessi and Zonca 2019](#);

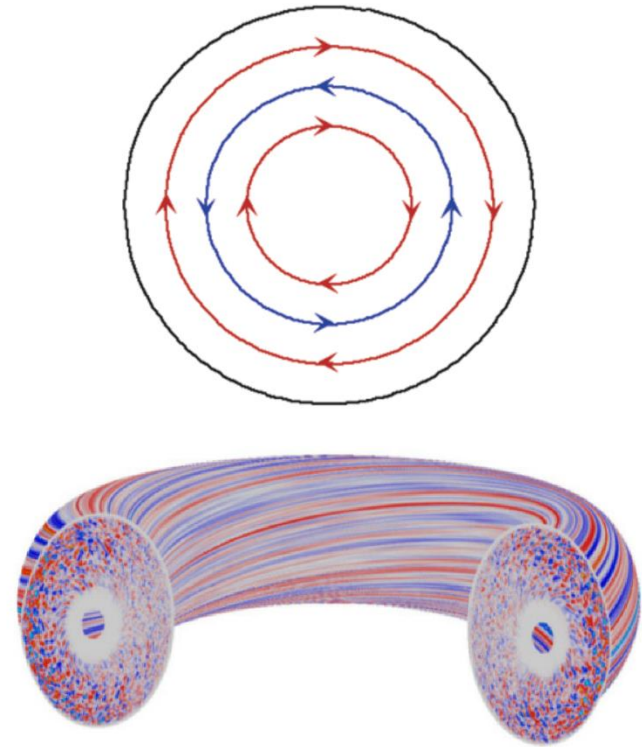


Figure: Courtesy of Y. Xiao et al., PoP 2015.



- Particle motion in the reference magnetic field is characterized by **three integrals of motion**, i.e. P_ϕ, μ, \mathcal{E} ;
- **Phase Space Zonal Structures** equation is connected with the **macro-/meso- scopic component**, i.e. $[\dots]_S$, unperturbed orbit averaged distribution function (Falessi and Zonca 2019);

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \overline{(\tau_b \delta \dot{P}_\phi \delta F)}_z + \frac{\partial}{\partial \mathcal{E}} \overline{(\tau_b \delta \dot{\mathcal{E}} \delta F)}_z \right] = \left(\sum_b C_b^g [F, F_b] + \mathcal{S} \right)_{zS}$$

- This expression describe **transport processes in the phase space** due to fluctuations, collisions and sources;



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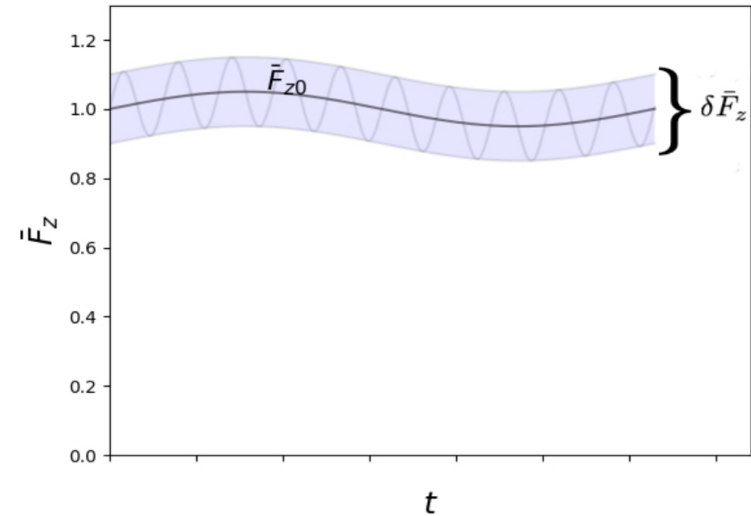
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- This expression describe **transport processes in the phase space** due to fluctuations, collisions and sources;
- Transport equations can be obtained by averaging in the velocity space [Falessi and Zonca 2019](#);
- **mesoscales** are spontaneously created by the turbulence;



- Having defined **Phase Space Zonal Structures**, we can decompose the **toroidally symmetric distribution function**;
- $\overline{F_{z0}}$ describe **macro- & meso-scales**;
- **micro-scales** are accounted by $\delta\overline{F_z}$;

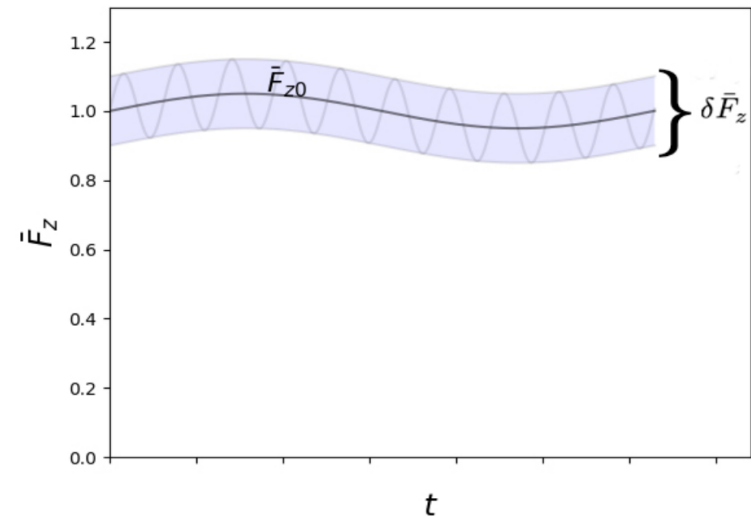
$$F_z = \overline{F_{z0}} + \delta\overline{F_z} + \delta\tilde{F}_z$$





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- $\overline{F_{z0}}$ describe **macro- & meso-scales**;
- **micro-scales** are accounted by $\delta\overline{F_z}$;
- they describe system transitions between **neighboring nonlinear equilibria**, see [Chen and Zonca 2007](#); [Falessi and Zonca 2019](#);
- **nonlinear equilibria**, together with **zonal fields**, form a **zonal state**.

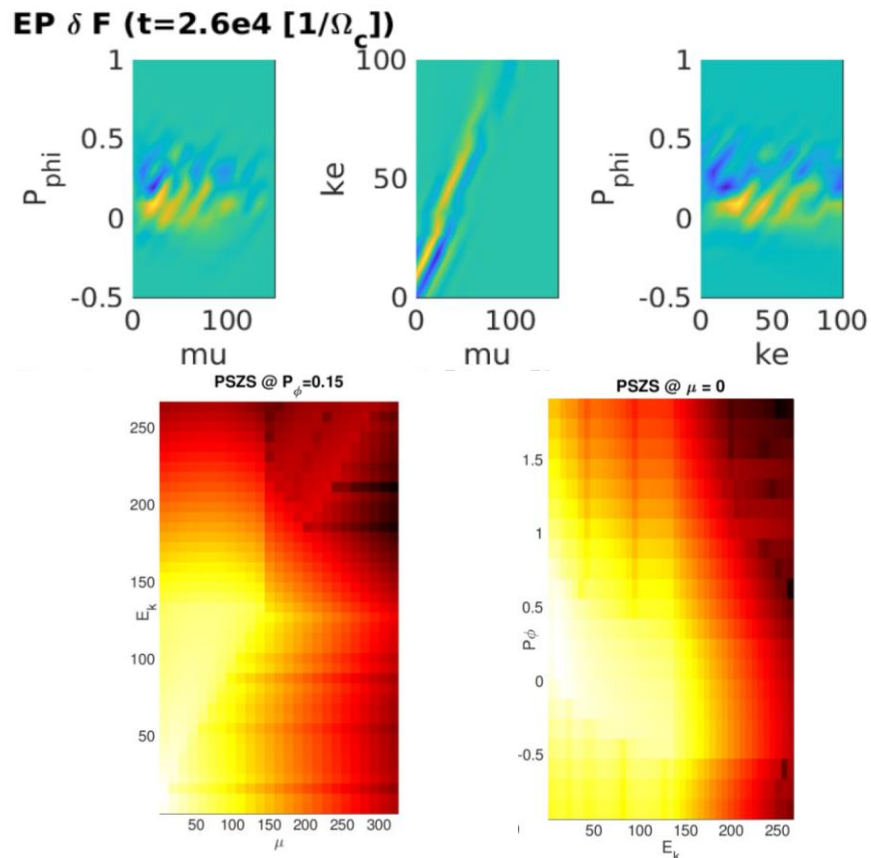
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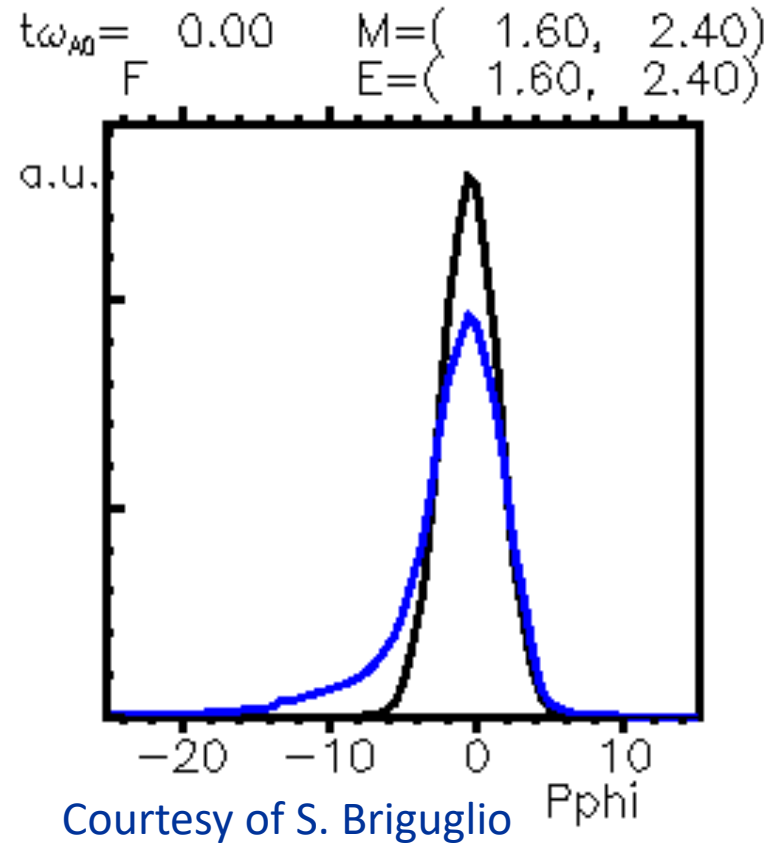
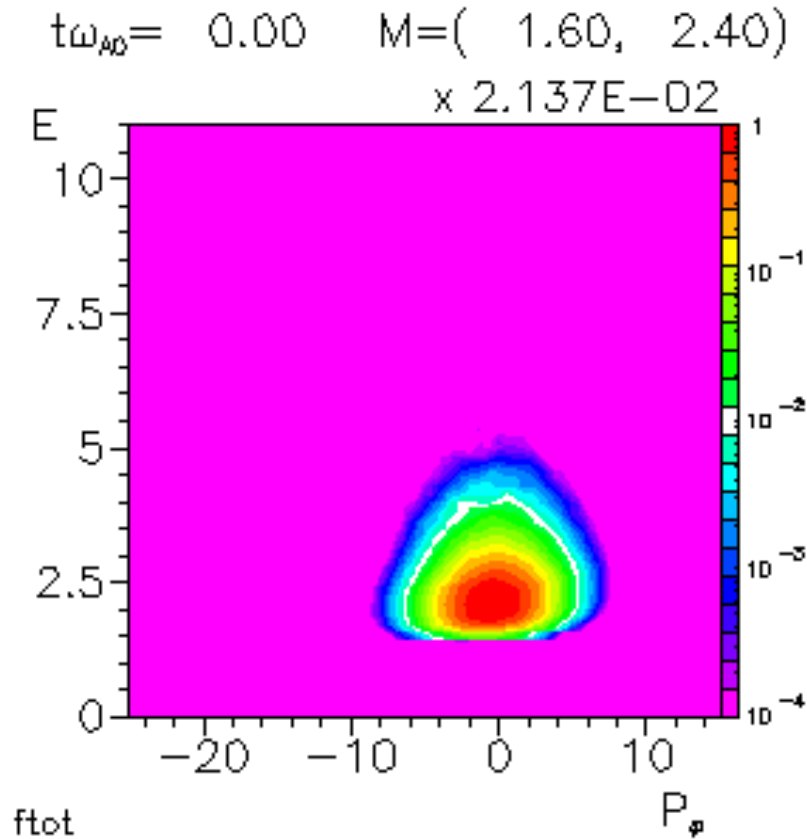
Phase Space Zonal Structures diagnostic in ORB5



- an **ORB5** diagnostic for **PSZS** has been developed, i.e., see [Bottino et al \(2022\)](#);
- **PSZS** can **accumulate** over time...
- A restart of **ORB5** from **PSZS** data is the next step, see the contribution by [A. Bottino at EPS2023](#);
- Illustration of ORB5 application to frequency chirping modes ([Invited Talk by X. Wang at EPS2023](#))



PSZS diagnostic in HMGC





- PSZS transport is studied by means of a **hierarchy of verified and validated reduced models** within an **EUROfusion Enabling Research Project** with **P. Lauber** as P.I.;
- explicit expression of **EP fluxes in PSZS** equations have been calculated within the following hierarchy of simplifying assumptions:
 - the **zeroth level of simplification** consist in the gyrokinetics description of plasma dynamics;
 - the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, [Zonca et al 2021](#);
 - the second and final level of simplification is the **quasilinear model**



DAEPS (Y. Li et al 2020)

- **Ballooning decomposition** for fluctuations;
- Based on **fish-bone like dispersion relation**;
- Mode structure decomposition, **separation of radial envelope and parallel mode structure**;
- Calculate nonlinear fluxes by the **DSM model** or a saturation rule.

LIGKA-HAGIS (Lauber et al 2007)

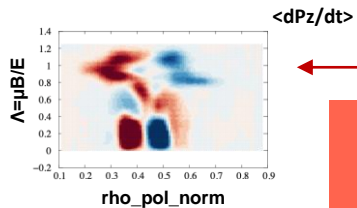
- **Fourier decomposition** for fluctuations;
- Solve **linear gyrokinetic equation**;
- Assume fluctuations amplitude, e.g., kick-model or quasilinear;
- Use **IMAS-coupled EP stability WF (HAGIS/LIGKA)** to **calculate Phase Space Zonal Structures fluxes**.

ATEP: kick model limit

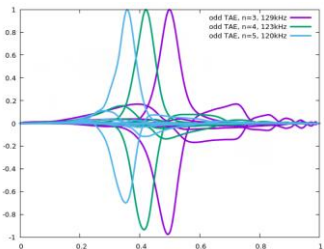


ATEP code: solve transport equation for PSZS with sources and collisions, [Lauber 2022](#)

$$\frac{\partial}{\partial t} F_{z0} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} (\tau_b \delta \dot{P}_\phi \delta F) \right]_z + \frac{\partial}{\partial \mathcal{E}} (\tau_b \delta \dot{\mathcal{E}} \delta F) \Big|_S = \left(\sum_b C_b^g [F, F_b] + S \right) \Big|_{zS}$$



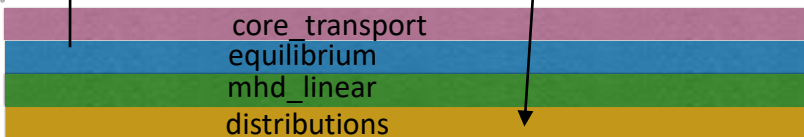
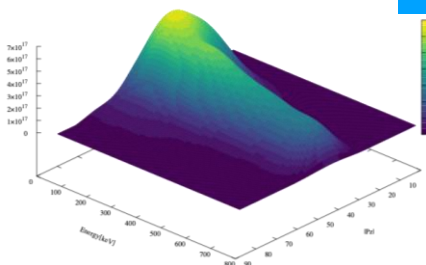
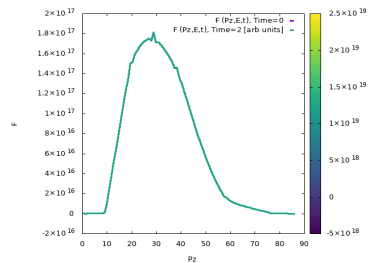
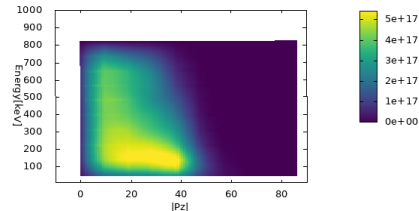
calculate PSZS fluxes with prescribed amplitude



calculate linear mode spectrum

DAEPS and LIGKA Are interchangeable Thanks to IMAS

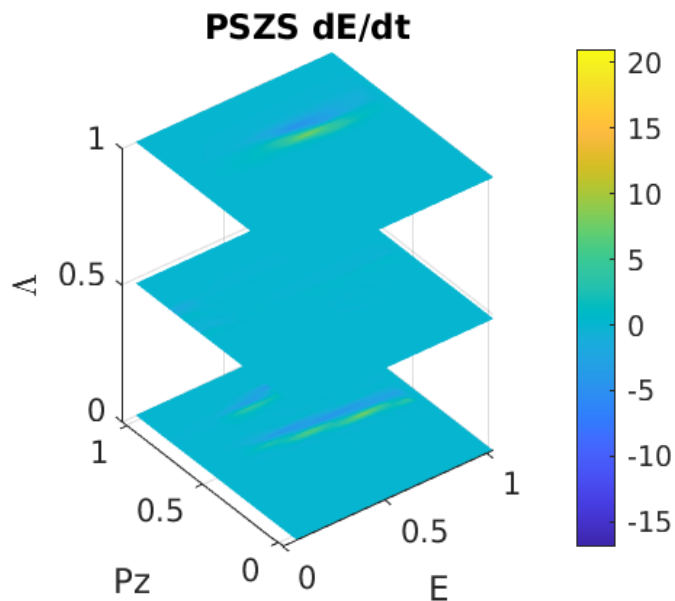
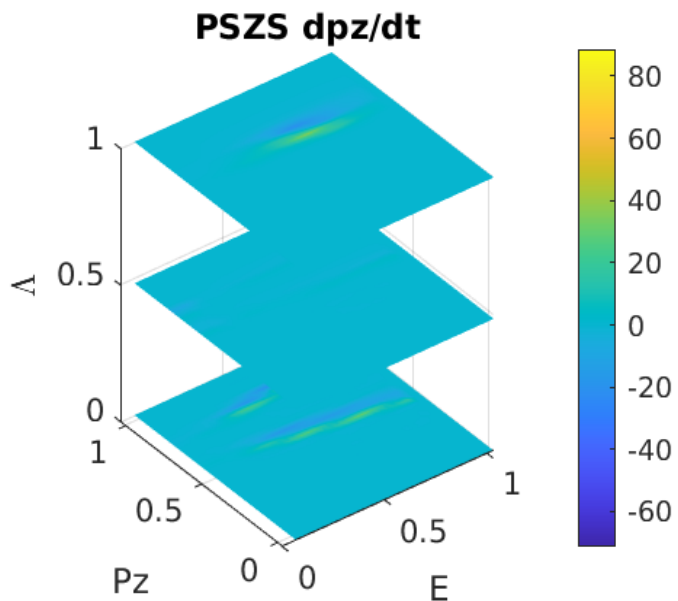
advance F_{EP} and return updated F_{EP} into IDS, or its moments



time



$$\frac{\partial}{\partial t} \overline{F_{z0}} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \overline{(\tau_b \delta \dot{P}_\phi \delta F)}_z + \frac{\partial}{\partial \mathcal{E}} \overline{(\tau_b \delta \dot{\mathcal{E}} \delta F)}_z \right]_S = \left(\sum_b C_b^g [F, F_b] + S \right)_{zS}$$

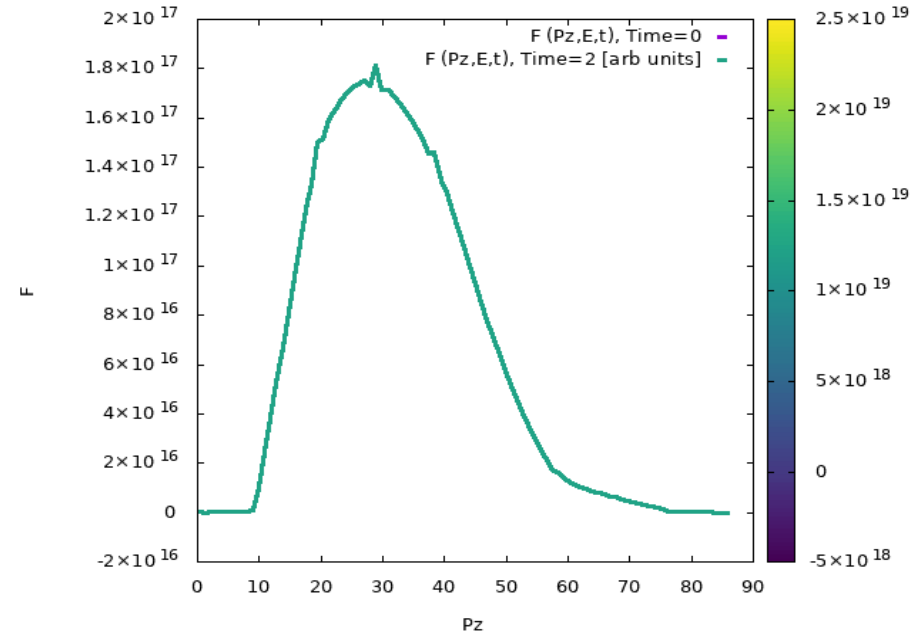
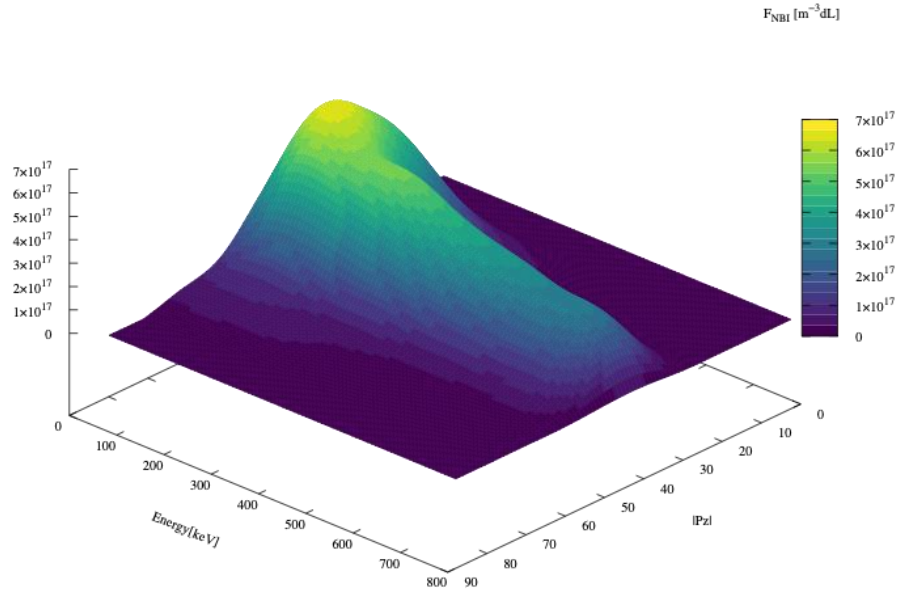


G. Meng et al. 14th International West Lake Symposium



ATEP code: solve transport equation for PSZS with sources and collisions, [Lauber 2022](#)

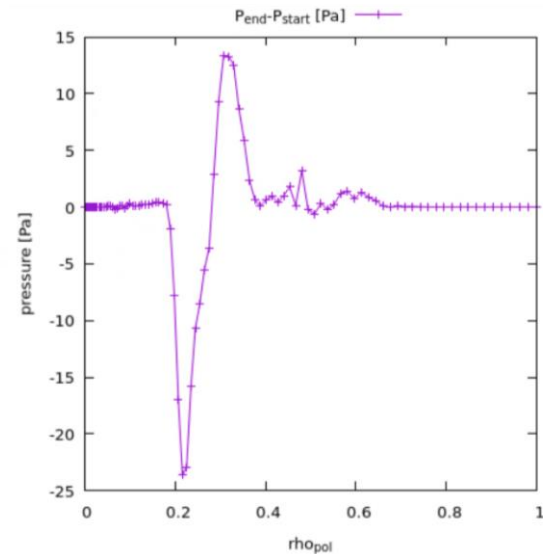
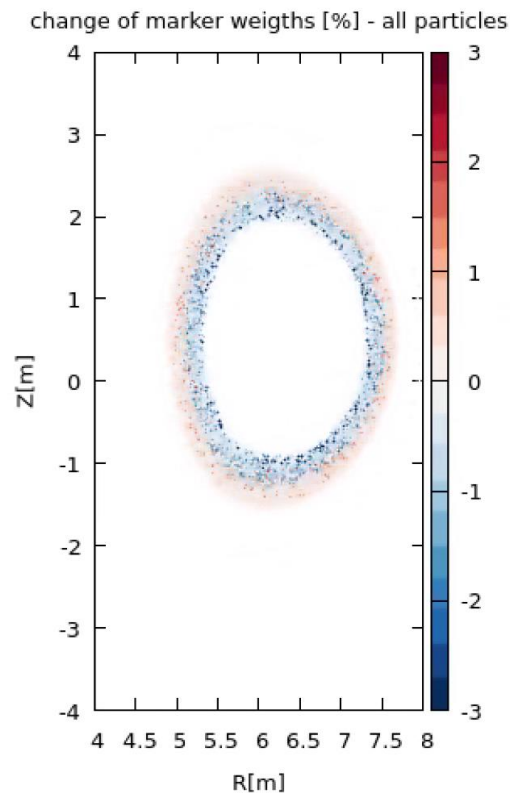
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ITER plasma from H&CD WF by M. Schneider



- Mapping to 1d profile of **PSZS** is possible;
- a **CGL equilibrium** can be readily constructed [Falessi et al 2023 sub](#);
- transport is **zonal** by construction;
- Next step is the calculation of the corresponding **magnetic equilibrium**;

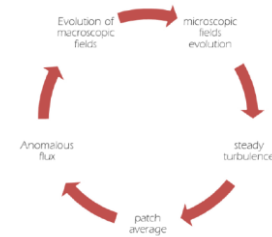


ATEP code: solve transport equation for PSZS with sources and collisions, [Lauber 2022](#)

Summary & Conclusions



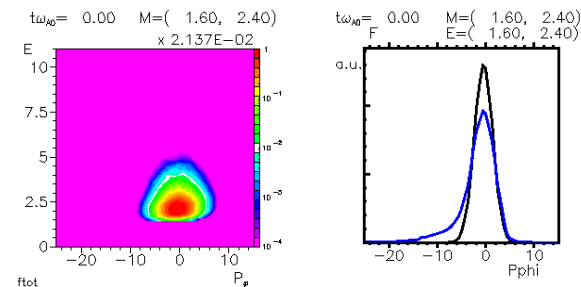
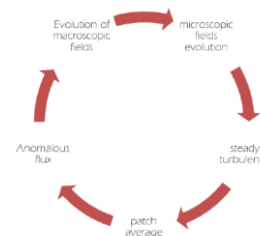
- Need of **predictive transport models** on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing **EPs**;



Summary & Conclusions



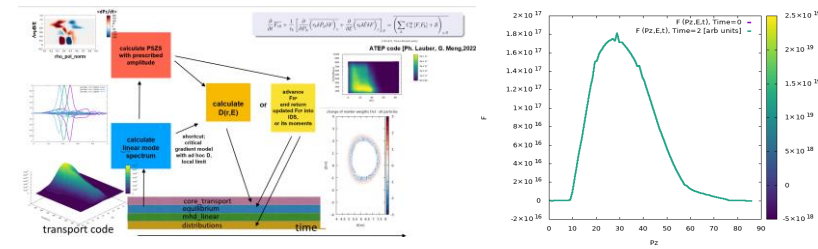
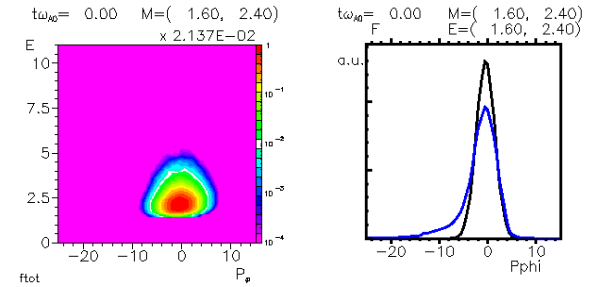
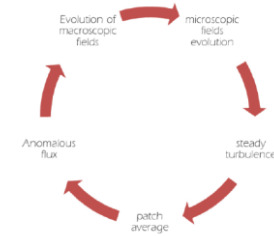
- Need of **predictive transport models** on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing **EPs**;
- we have shown how to describe **meso-scales** and **non-Maxwellian distribution functions** using appropriate phase transport equations;
- we have introduced the concept of **zonal state** to describe the evolution of the plasma between neighboring nonlinear equilibria;



Summary & Conclusions



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- we have shown how to describe **meso-scales** and **non-Maxwellian distribution functions** using appropriate phase transport equations;
- we have introduced the concept of **zonal state** to describe the evolution of the plasma between neighboring nonlinear equilibria;
- we have introduced a conceptual framework which allow to **study EP phase space transport** over long time scales.

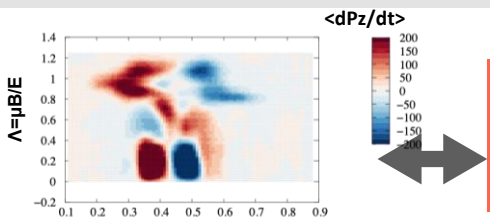


ATEP: Quasilinear model

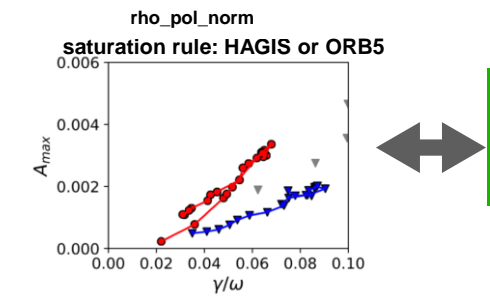


ATEP code: solve transport equation for PSZS with sources and collisions [Lauber 2022](#)

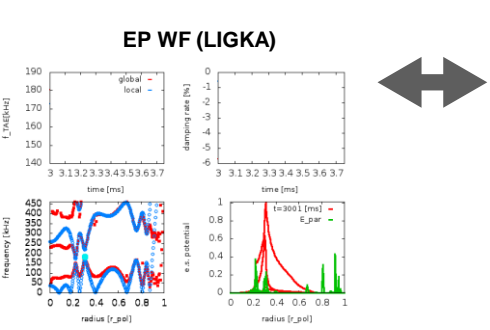
$$\frac{\partial \overline{F_{z0}}}{\partial t} + \frac{1}{\tau_b} \left[\frac{\partial}{\partial P_\phi} \left(\tau_b \delta \dot{P}_\phi \delta F \right) \right]_z + \frac{\partial}{\partial \mathcal{E}} \left(\tau_b \delta \dot{\mathcal{E}} \delta F \right) \Big|_S = \left(\sum_b C_b^g [F, F_b] + S \right) \Big|_{zS}$$



calculate PSZS



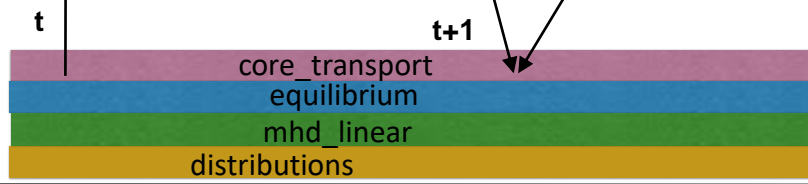
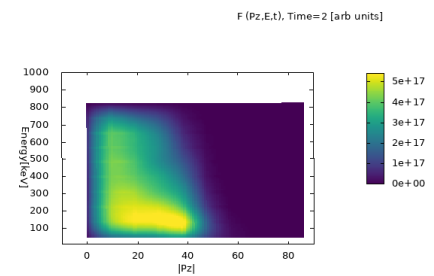
use NL code/model for intensity closure



calculate linear mode spectrum

calculate D(r,E)

or
advance F_EP and return updated distribution IDS, or its moments





- We can re-write the **low frequency (transport) component** of δf_z in term of the PSZS;
- taking the time derivative of the surface averaged velocity integral we obtain:

$$\partial_t \langle \langle \delta f_z \rangle_v \rangle_\psi = \frac{e}{m} \left\langle \left[1 - \overline{(e^{-iQ_z J_0})} \overline{(e^{iQ_z J_0})} \right] \frac{\partial F_0}{\partial \varepsilon} \partial_t \delta \phi_z \right\rangle_v + \frac{1}{v'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \overline{(e^{-iQ_z J_0})} \left[c e^{iQ_z R^2} \nabla \phi \cdot \nabla \langle \delta L_g \rangle \delta G \right] \right\rangle_v \right\rangle_\psi$$

- This equation describes the **radial oscillations on any length-scale** of the density profile in the absence of collisions and assuming GK ordering [Falessi and Zonca 2019](#);
- **mesoscales** are spontaneously created by the turbulence;



- the current \mathbf{J}_z and the pressure tensor \mathbf{P} satisfy the following force balance equation:

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_{\parallel} + (\sigma - 1) \nabla \left(\frac{B^2}{8\pi} \right) + \frac{B^2}{4\pi} \nabla_{\perp} \sigma$$

where $\sigma = 1 + \frac{4\pi}{B^2} (P_{\perp} - P_{\parallel})$;

- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

$$\Delta^* \psi + \nabla \ln \sigma \cdot \nabla \psi = - \frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2} \frac{\partial G}{\partial \psi}$$

- where $G \equiv \sigma F^2 / 2$ is a flux function;



- EPs are generally non Maxwellian, and transport processes take place in the phase space!
- Separation of scales is questionable...
- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;



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- Separation of scales is questionable...
- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;
- interaction with thermal plasma over long timescales can modify bulk transport processes;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the theory of Phase space zonal structures (PSZS), see Chen and Zonca 2016.